Current Research in Ohio Archaeology 2025
www.ohioarchaeology.org

The Geometry of the Newark Earthworks

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Abstract

In their 1982 paper, *Geometry and Astronomy in Prehistoric Ohio*, Drs. Ray Hively and Robert Horn of Earlham College noted several relationships involving a similarity of area between the square and circular enclosures of the Newark, Ohio earthworks. They and other authors have also proposed that one or more units of measurement may have been employed in their layout and construction. This paper attempts to (1) present data in support of these assertions; (2) show how the builders may have conceptualized circles and squares of equal area; (3) suggest how they might have gone about achieving these complex relationships in their monumental earthen constructions; and (4) how a particular system of measurement might have facilitated this.

Introduction

"Hopewell" is the name given to a group of pre-Columbian peoples whose sphere of influence, at its apex some 2,000 years ago, extended throughout much of what is now the midwestern United States, principally along the Mississippi and Ohio Valleys. Despite an abundance of artifacts and skeletal remains, many questions remain unanswered. One prominent legacy of the Hopewell is a number of earthen mounds and geometric enclosures, some of monumental proportions. Some of these structures have survived (Figure 1a), notably in the area in and around what is now Newark, Ohio, which is believed to have once been an important center of exchange and social or ceremonial activity (Lepper 1995:56).

One intriguing question arising from a careful examination of the enclosures at the Newark site is whether the builders may have been concerned with constructing squares and circles of similar area and if, in the process, they might have found a practical but very accurate solution to a problem that occupied the minds of Old World mathematicians for nearly 4000 years: that of "squaring the circle."[1]

History

The five earthen enclosures known today as the Newark Earthworks—three of them circular in shape, one octagonal, and the fifth a fragment of what was once a square—are virtually all that remain of a much larger complex of tombs, mounds, walled roads, and geometric structures (Figure 1b). Their first systematic description dates from 1848, when Ephriam Squier and Edwin Davis published *Ancient Monuments of the Mississippi Valley*, a work now valued more for the qualitative documentation provided by its illustrations than for the accuracy of the surveys undertaken by its authors. The roads and canals of the early European

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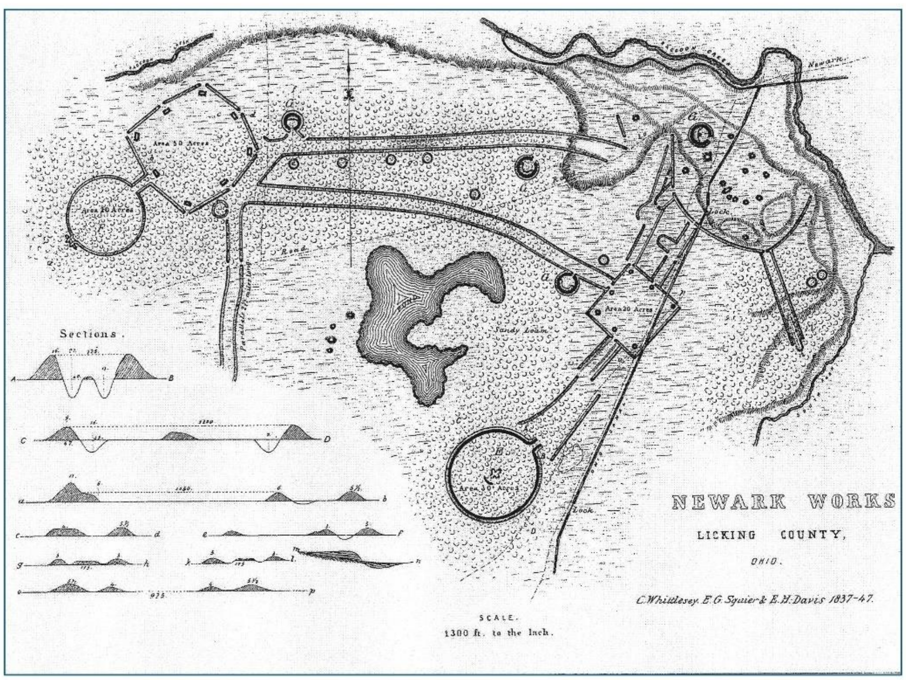

b

Figure 1. a, Aerial view of the Observatory Circle and Octagon earthworks in Newark, Ohio, Friday afternoon, Jan. 25, 2008. (Photo by Timothy E. Black/Newark Earthworks Center)**; b, Map of the Newark Earthworks.** (From *Ancient Monuments of the Mississippi Valley*, Squier and Davis 1848)**.**

settlers, which appear only incidentally on Squier's and Davis's drawings, presage Newark's industrial and commercial transformation in the nineteenth century, a process which would subsequently destroy much of what they had seen and recorded. Nor have the five surviving structures been immune to the ravages of change: Much has been destroyed or intentionally altered (some of the changes coming in the form of well-meant "restoration"), making an accurate determination of their original features somewhat less than straightforward.[2]

In 1881, the United States Congress commissioned the Smithsonian's Bureau of Ethnology to undertake a study of what then remained of the Moundbuilders's culture. The resulting report of what has been called "the first modern archaeology carried out in America" (Smith 1985:5) was prepared by the head of the bureau's Division of Mound Exploration, Cyrus Thomas, and published in 1894. Thomas's descriptions and detailed surveys in some places corroborate, but frequently correct,[3] the earlier work of Squier and Davis. It is largely because of the Bureau's investigations that we have reliable information not only about the Newark works, but of many others of which little or no trace remains today.

Then in 1982, Drs. Ray Hively and Robert Horn, professors of physics and philosophy, respectively, at Earlham College, published a paper whose findings would seem to underscore the importance of this extensive earlier documentation. In *Geometry and Astronomy in Prehistoric Ohio*, Hively and Horn, relying on data from the Bureau's report and from their own surveys, argue that the Hopewell may have incorporated a number of astronomical alignments into the design of the Newark Earthworks. Remarkably, an observer standing at specific locations within the earthworks is able to witness the eight extreme rise and set points of the moon during an 18.6-year cycle.

Internal Geometry

Perhaps no less remarkable are the mathematical relationships that may exist between the geometric enclosures themselves. Of the surviving five, the larger four are known as the Great Circle (sometimes called the Fairground Circle); the slightly smaller Observatory Circle, so named because of a large mound (the Observatory Mound) located at a strategic point on its circumference; the Octagon, which is connected to the Observatory Circle by an avenue bordered by two parallel walls; and the Newark Square, of which only a corner exists today. The fifth enclosure, a much smaller circular enclosure adjacent to one of the sides of the Octagon, has never been given a proper name, perhaps because of its size relative to the larger figures. But despite being almost overlooked in historical accounts, it plays a crucial role in the system of lunar alignments that Hively and Horn argue are incorporated into the design of the earthworks, and as will be seen later in the discussion of the Octagon, it may play a similarly important part in the geometry of the earthworks as well.

Hively and Horn (1982:S20) have proposed that the four larger figures are related as follows (Figure 2):

1. The area of the Newark Square is the same as that of the Observatory Circle.
2. A square whose side is equal to the diameter of the Observatory Circle (i.e., the square which would circumscribe the Observatory Circle) has the same area as the Great Circle.[4]

3. The area of the Octagon is twice the combined areas of the Observatory Circle and the avenue which connects them.

Additionally, William Romain states that

4. The perimeter of the Newark Square is equal to the circumference of the Great Circle.[5]

These relationships are by no means trivial. And if they could be verified with reliable data, it would suggest that the Hopewell possessed a considerable level of mathematical sophistication; specifically, that the Hopewell had a practical knowledge of the ratio of a circle's circumference to its diameter (what we know as π, or *pi*); that they could compute the areas of both squares and circles; and that they may have devised a method of accurately constructing squares and circles of equal areas.

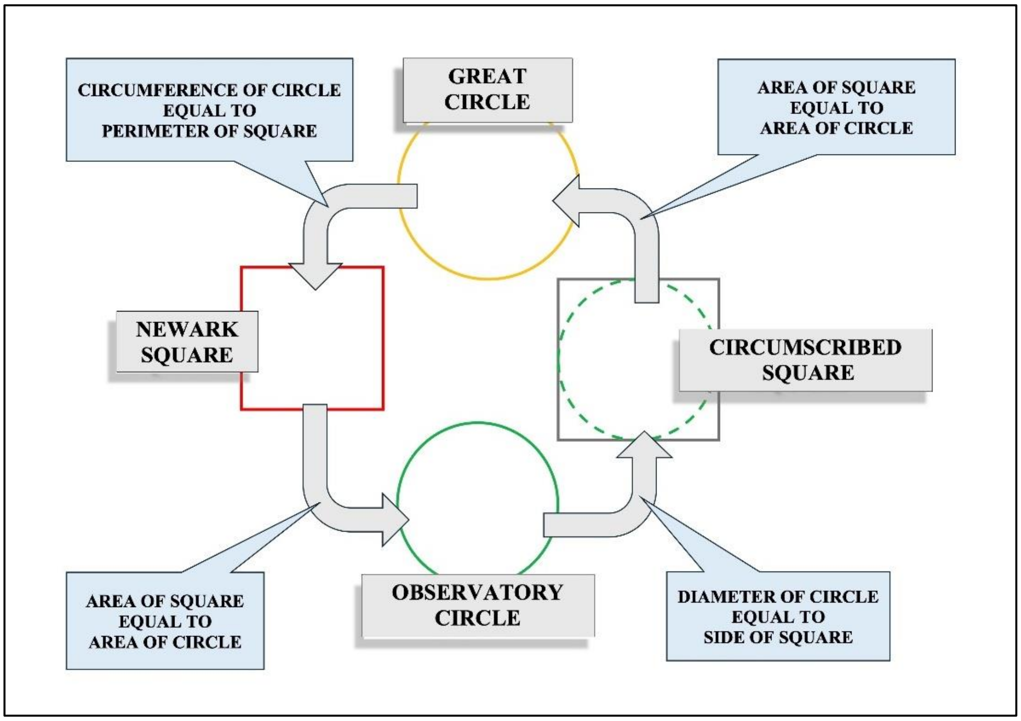

Figure 2. The proposed geometric relationships between the Newark enclosures.

Site Evidence

Whether or not these relationships exist depends on how closely the relative dimensions of the enclosures approximate those we would expect. Of the dimensions of the four major figures, those of the Observatory Circle (Figure 3a) are perhaps the most reliable and least ambiguous in that it is nearly a perfect circle,[6] and Middleton's value for its diameter of 321.3 m (1054 ft) is in precise agreement with those of subsequent surveys.[7] The surveyed diameter also appears elsewhere in the Newark works and at other Hopewell sites as well. For these reasons, it is chosen here as a standard of comparison.

Unlike the Observatory Circle, the Great Circle (Figure 3b) is somewhat less than perfect (Middleton noted a difference of 7.9 m [26 ft] in its maximum and minimum widths). But Hively's and Horn's estimate of its diameter (361.2 m), which they based on their determination of the true circle providing the "best fit," deviates from the diameter of the circle having the ideal circumference and area values by less than 1.4 meters, or 0.4 per cent, and is only slightly smaller than the enclosure's maximum width of 362.4 m (1189 ft) as reported by Middleton.[8]

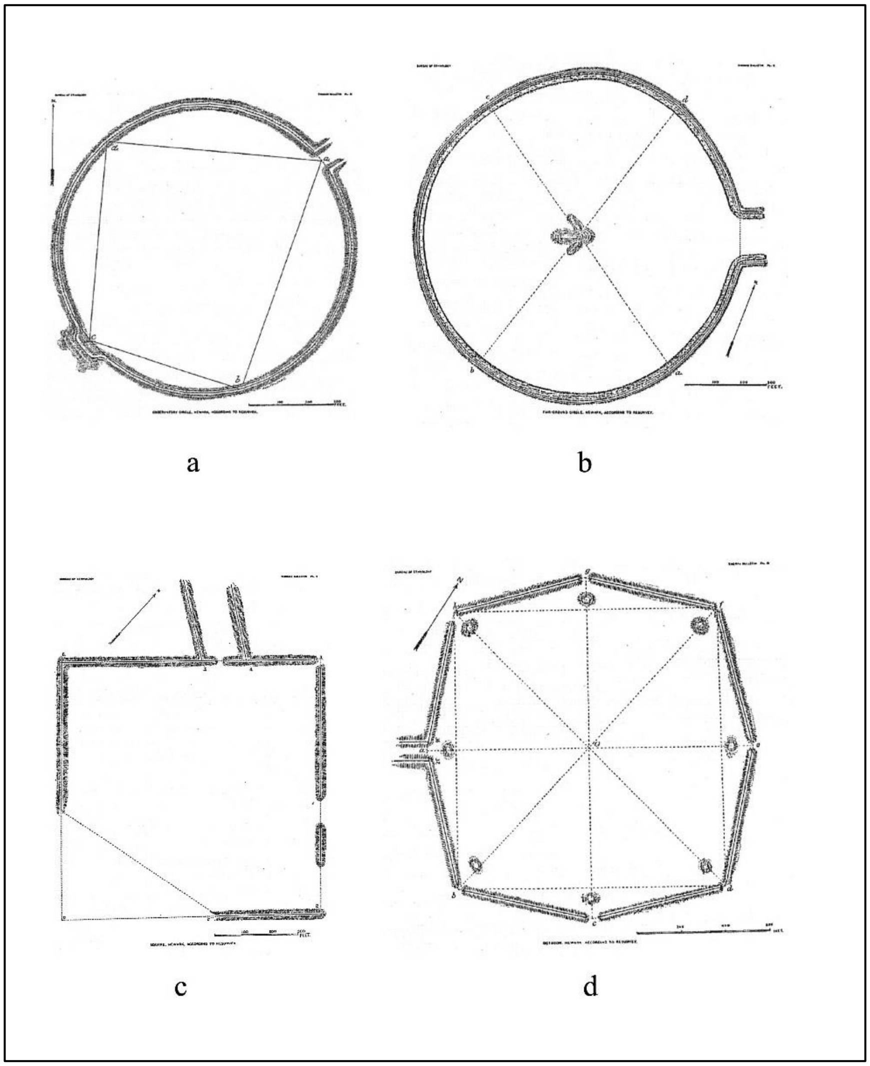

Figure 3. a, The Observatory Circle; b, The Great Circle; c, The Newark Square; d: d, The Octagon. (From Thomas 1894)**.**

The Square (Figure 3c) is quite regular: Half of it is nearly a perfect square, its two adjacent walls of 282.2 m (926 ft) and 282.9 m (928 ft) intersecting at an angle which is within twenty minutes of ninety degrees. The other two walls, estimated at 283.8 m (931 ft) and 286.2 m (939 ft), are slightly less than a degree out of square, as is an adjacent angle.[9] The Square's longest side differs by only 0.8% from the average of the sides, but given the Hopewell's evident ability to construct right angles with considerable accuracy[10], this may be noteworthy; even more so, in that there are squares of remarkable regularity at other Hopewell sites in Ohio, specifically the Baum and Liberty Earthworks (Thomas 1894:482-483). But the average of Middleton's measurements for the four sides differs by about one meter from the length of the side needed for the Square's area to be the same as that of the Observatory Circle, and it is precisely the length required for actual perimeter-circumference equality with the Great Circle. [11] If we accept the diameter of the Observatory Circle as very close to the one intended, then using it as a standard of comparison, we find that the surveyed dimensions of the Great Circle and Square are quite close to what we would expect if the area and perimeter relationships existed (Figure 4).

The Problems

The area relationship between the Observatory Circle and the Newark Square was the goal of an ancient problem of geometric construction known as the squaring (or quadrature) of the circle. The task before the geometer was to construct, using only straightedge and compass, a square having the same area as a given circle. Since it is quite likely that a length of cord or rope[12] fixed at one or both ends and drawn tight, the equivalent of compass and straightedge (respectively), was used in the laying out of the enclosures, a consideration of the problem in terms of its classical restrictions seems relevant here. But if the architects of the earthworks were limited by the rules of classical geometric construction, the apparent correspondence of the Newark figures would have been quite difficult to achieve. The circumference-perimeter problem may help illustrate the difficulties the Hopewell would have faced and how they might have overcome them.

Rectifying the Circumference

The act of constructing a square whose perimeter is equal to the circumference of a given circle is sometimes referred to as the rectifying of the circumference. In this case, if we let d represent the diameter of the Great Circle (giving it a circumference of πd), and p the perimeter of the Newark Square, then

$$p = \pi d$$

Thus, the ratio of the length of the perimeter of the square to the diameter of the circle must be equal to π. The difficulty here is that because π is what is known as a transcendental number, it is not possible, using only straightedge and compass, to construct a straight line whose length is π times the length of a given line.[13] The builders, then, must have employed some other method.

So how might they have gone about it? The answer may have partly to do with the scale in which the builders worked. In the preliminary "laying out" of a circular enclosure, it is likely that the Hopewell erected wooden posts along the circumference to indicate the location of the earthen walls.[14] In the case of the Great Circle, if these posts were placed 10 meters apart, the inscribed polygon that would be formed by connecting them with straight lines would have 113 sides and its perimeter would approximate the circumference within about 4.7 m (0.4%).[15]

One could also determine the perimeter of the Square by laying a very long cord or rope along the circumference of the Circle. But this method only works in one direction (try forming a circle with a length of string); nor would it be useful in creating the area relationships between

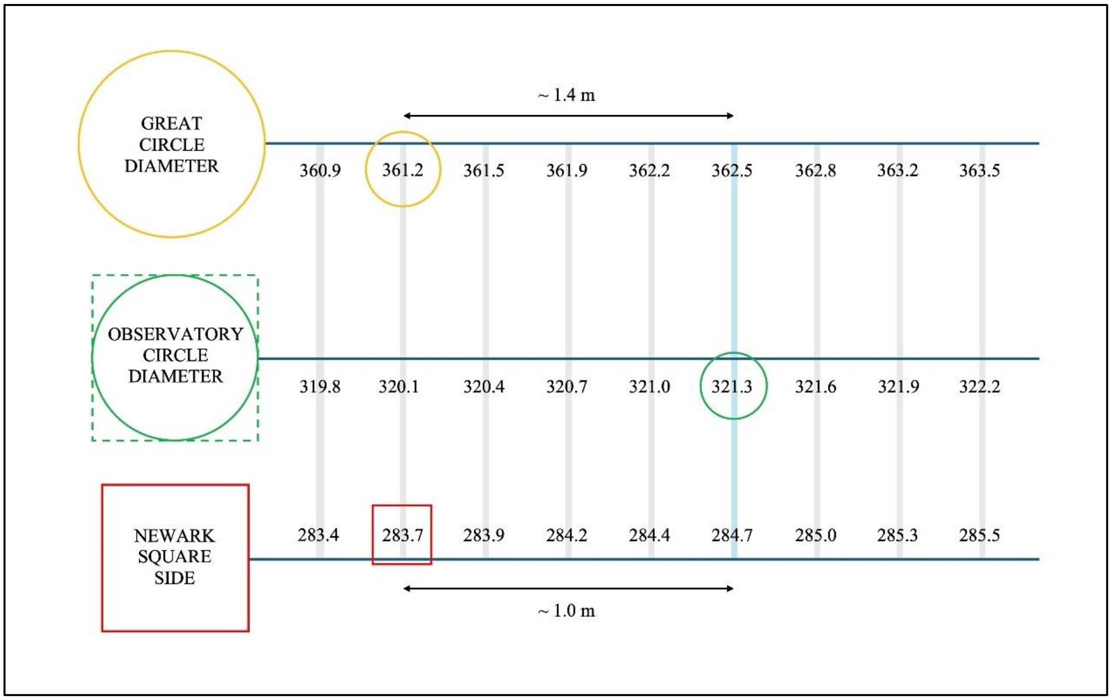

Figure 4. The ideal dimensions (in meters) of the Great Circle and Newark Square as they would correspond to a range of values for the diameter of the Observatory Circle.

the enclosures. More generally, though, using a cord or rope the length of the *diameter* of the circle would have enabled the Hopewell to arrive at a very accurate approximation of the ratio of the circumference of *any* circle to its diameter—what we call π—in a very simple way: Take a cord equal to the diameter and carefully lay it along the circle, noting how many whole lengths fit along the circumference (three, the integral part of π). Then lay the cord on what remains of the circle and fold the cord repeatedly upon itself in lengths equal to this remainder (essentially dividing the diameter by what is left of the circumference to obtain the fractional part of π, which is approximately one-seventh). Adding the two, we arrive at a figure of $3^1/_7$ diameters. $3^1/_7$, or $^{22}/_7$, is an excellent practical approximation[16] of π: For a circle 321 m in diameter, it yields a value for the circumference with an error of less than 0.4 m. And while this procedure may be somewhat lacking in mathematical sophistication, it is worth noting that the result is a direct and concrete representation of the circumference-diameter relationship and is independent of any number system or particular unit of measurement (Figure 5).

It may be significant in this context that the diameter of the small circular enclosure adjacent to the Octagon is almost exactly one-seventh the diameter of the Observatory Circle (Hively and Horn 1982:S9). Hively and Horn, in arguing that "the length of 1 OCD had a special significance for the builders of the earthworks", note that the distance between the centers of the Observatory and Great Circles and the distance between the centers of the Octagon and the

Square are each equivalent to six times the OCD.[17] The relative locations of these figures would presumably have been determined by successively marking off segments equal to the OCD along a line between them. If three such segments were joined in this way to a segment equal to the diameter of the smaller mound ($^{1}/_{7}$ OCD), the total distance would equal $^{22}/_{7}$ OCD --- the circumference of the observatory circle (Figure 6).

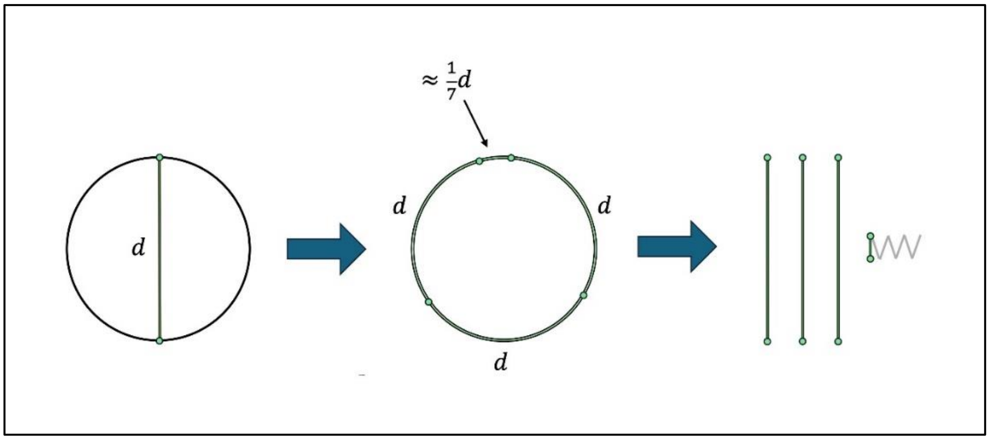

Figure 5. Practical method of approximating π with cord or rope.

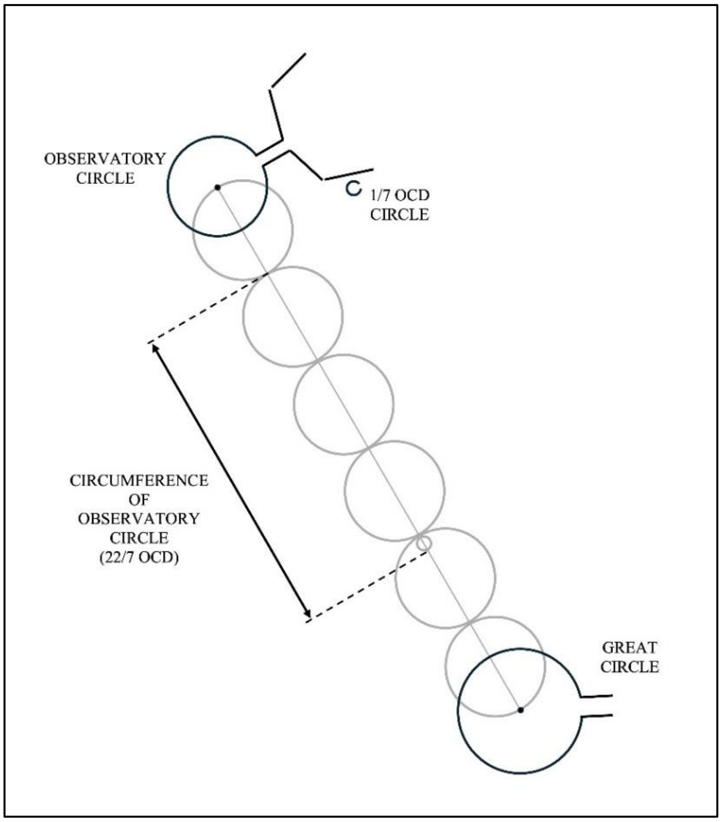

Figure 6. The circumference of the Observatory Circle is constructed from multiples of the OCD and the diameter of the small circle. The illustration is adapted from one which appears in Hively and Horn (1982) and Lepper (1998:8).

There are good reasons for thinking that $^{22}/_{7}$ might have been the likely approximation for π, if one was used: (l) It's simple; (2) it's very close to the actual value; (3) it's easily found by the method described; and (4) the scale in which the builders worked would have easily allowed for an approximation of this accuracy. And by constructing a line segment equal to 1/7 of its diameter, the circumference of any circle could be easily determined.

Squaring the Circle

We can define the problem of squaring the circle in the same way that we defined it for the circumference-perimeter relationship: If a square of side *s* and a circle of radius *r* have the same area, then

$$s^2 = \pi r^2$$

which, after taking the square root of both sides, becomes

$$s = r\sqrt{\pi}$$

which tells us that in order to construct the Newark Square, the planners this time would have had to be able to produce a length whose ratio to the length of the radius of the circle is equal to the *square root* of π, which is also impossible under the rules of classic construction.[18] But more importantly, unlike in the case of circumference-perimeter correspondence, there is no simple empirical method of determining the relative linear dimensions of circles and squares having the same area.

It might be helpful at this point to put the matter into some sort of conceptual perspective by looking at how other early cultures possessing some approximation of π might have envisioned the translation of circular areas into rectangular ones. One traditional explanation involves dividing a circle into pie-shaped sections and then fitting the pieces together (Figure 7):[19] The greater the number of sections into which the circle is divided, the more closely the assemblage approximates a true rectangle of sides πr and r, which is then seen as having the same area (πr^2) as a circle of radius r. But there is no straightforward method of converting such a rectangle into a square.

It does not seem unreasonable to suggest that the builders might have thought about the area of a circle in terms of its transformation into a rectangle (an argument for this will be presented later in the discussion of the Octagon). But it is possible that they might have arrived at the dimensions of the enclosures by discovering a relationship between lengths and areas which is simple, direct, and perhaps unique. As noted previously, evidence from Newark and from other sites strongly suggests that the diameter of the Observatory Circle, or some related unit of length, was used in the layout of the Newark Works. James A. Marshall, a civil engineer who has surveyed over two hundred Hopewell sites, maintains that there are not one, but *two* basic units employed in the layout of many of the earthworks (Marshall 1987). According to Marshall, the primary unit (which we'll call $\boldsymbol{u}$), common to over one hundred of the sites he surveyed, is approximately 57 m. The secondary unit (which we'll call $\boldsymbol{u_d}$) appears in some Ohio Earthworks, and at Newark in particular, and is 80.5 m, essentially one-fourth the OCD.[20] The two are

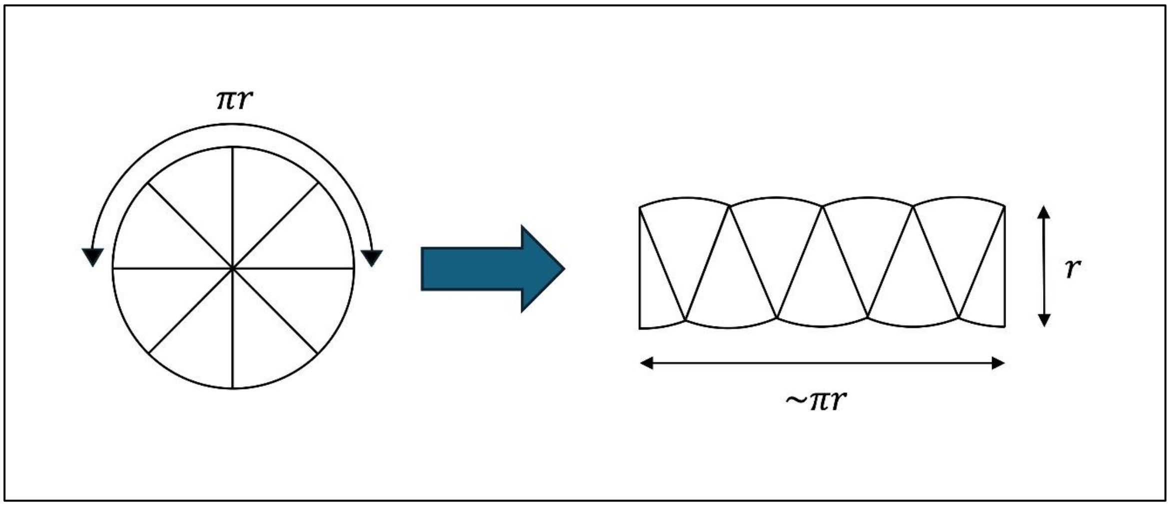

Figure 7. One way of conceptualizing the transformation of circular areas into rectangular ones.

related: 80.5 m is very close to 57√2 m, and is what Marshall describes as the "diagonal" of the basic unit. In other words, the length of the secondary unit ($\boldsymbol{u_d}$) is the diagonal of the square whose side is one basic unit ($\boldsymbol{u}$) long (Figure 8a).[21]

Marshall maintains that the Observatory Circle is laid out in diagonal units ($\boldsymbol{u_d}$), while the Newark Square is constructed using the basic unit ($\boldsymbol{u}$). The advantages of this dual system are apparent when a circle of diameter $4\boldsymbol{u_d}$ is superimposed over a square of side $5\boldsymbol{u}$ (Figure 8b): Visually, the areas of the square and circle appear similar, and in fact they are: The square is 5 basic units ($5\boldsymbol{u}$) on a side; the circle has a diameter of 4 diagonal units ($4\boldsymbol{u_d}$), which is equal to 4√2 basic units ($4\sqrt{2}\boldsymbol{u}$). Since the ratio of the side of the square to the diameter of the circle is 5 to 4√2, the side *s* of the square expressed in terms of the diameter *d* of the circle is:

$$s = \frac{5}{4\sqrt{2}}d = (.88388\ldots)d$$

which approximates the true value

$$s = r\sqrt{\pi} = \frac{\sqrt{\pi}}{2}d = (.88622\ldots)d$$

very closely: The length of side of the Newark Square computed from the two values differs by about 0.75 m. The parts of the square that lie outside the circle (shaded in the illustration) are quite close in area to the parts of the circle that lie outside the square: The area of the circle exceeds that of the square by only about 0.53 per cent.

If we look at Figure 8b strictly in terms of $\boldsymbol{u_d}$, it is apparent that a circle and a square will have approximately the same area if their diagonals (that is, the diameter of the circle and the diagonal of the square) are in a ratio of 4 to 5. Further, Marshall observes that many of the dimensions of the geometric figures at various sites, specifically those found in Ohio,[22] seem to be multiples of 4½ times the basic unit or its diagonal derivative (that is, multiples of $4\frac{1}{2}\boldsymbol{u}$ or $4\frac{1}{2}\boldsymbol{u_d}$). The ratio $4{:}4^1/_2$, or 8:9, is 0.88888...,[23] while the actual ratio of the diameter of the

Observatory Circle to that of the Great Circle is .88945... . A circle of diameter 4½u_d, or 4½ times the diameter of the Observatory Circle, would have a diameter of 361.4 m, essentially the same as Hively and Horn's figure for the diameter of the Great Circle. Thus, the Hopewell could have envisioned the dimensions of the square and circles in terms of a 4:4½:5 ratio (Figure 8c), or (more likely) one of simple integers: 8:9:10. In either case, by constructing squares and circles whose diagonal measurements are in simple ratios to one another, a square may be produced whose area is quite close to that of a given circle, and vice versa. A system of linear measurement would have enabled the Hopewell to quantify this relationship in an easy way and enable them to square any circle or circle any square with a remarkable degree of accuracy.

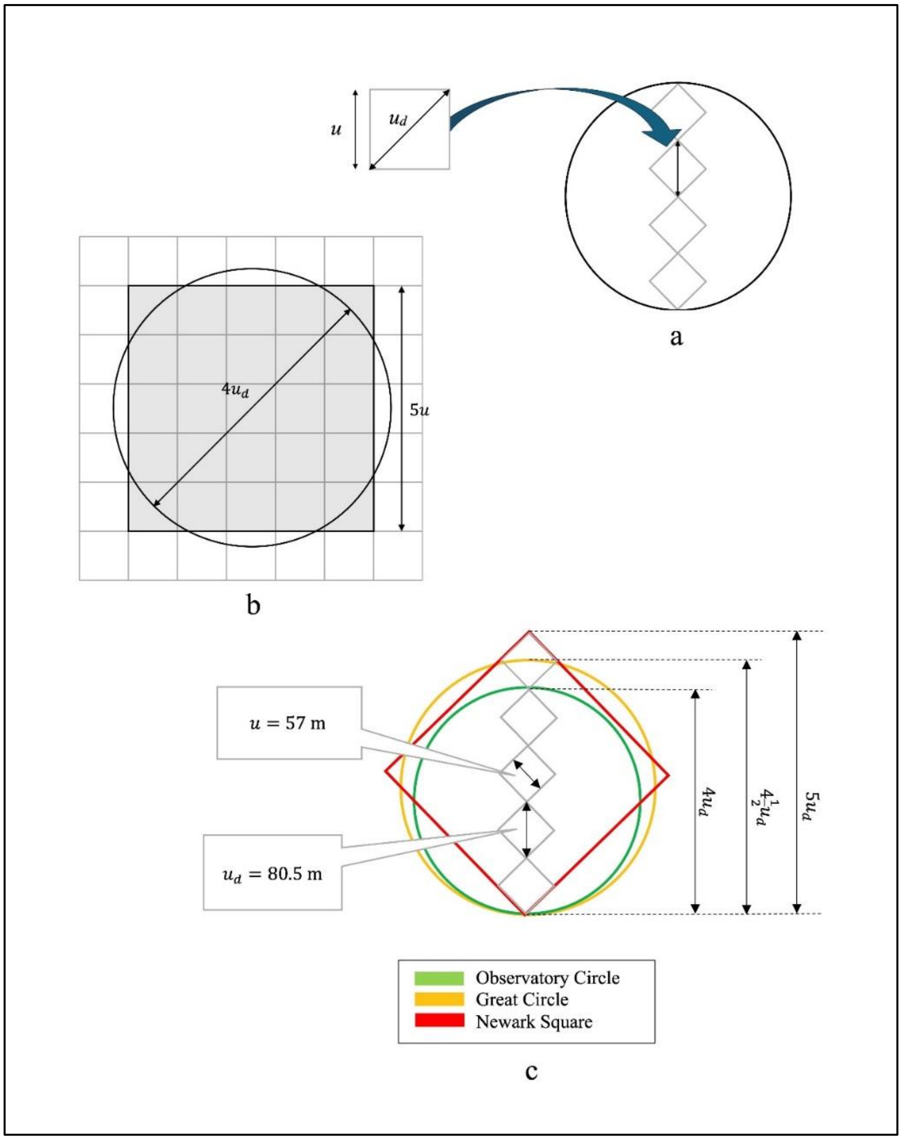

Figure 8. Marshall's Hopewellian units: a, The diameter of the Observatory Circle (OCD) as a multiple of the unit's "diagonal"; b, The Observatory Circle and Newark Square compared; c, The dimensions of the Newark enclosures as simple multiples of Marshall's units.

It is important to note that as a consequence of the area relationships between the Newark enclosures, the perimeter-circumference correspondences would occur incidentally—a sort of mathematical "bonus".[24] But this does not mean that the linear relationships were not intentional or that the builders were unaware of them.

The Octagon

In evaluating its usefulness as an astronomical instrument, Hively and Horn found themselves "unable to design an equilateral polygon with eight or fewer sides which incorporates the extreme lunar points more efficiently and accurately than does the Newark octagon" (Hively and Horn 1982). There is reason to believe that the Octagon (Figure 3d) may be remarkable from a strictly geometric standpoint as well.

Several authors (e.g., Fowke 1902:162; Hively and Horn 1982:S8) have described the likely method of the Octagon's construction: A circle the same size as the Observatory Circle is drawn, and a square is circumscribed about it. Circular arcs of radius equal to the diagonal of the square are then drawn from each corner of the square. The intersections of these arcs are then joined by lines to the corners of the square to form the perimeter of the Octagon (Figure 9a). The shape of the actual Octagon is virtually identical to one constructed in this manner. What minor deviations are present may be the result of an attempt *post factum* to improve the alignment of two of its sides (Hively and Horn 1982:S12).

Hively and Horn have also claimed that the area of the Octagon very nearly doubles the combined areas of the Observatory Circle and connecting avenue. If we compute the Octagon's area based on the ideal figure as constructed (before the presumed adjustments were made), we have

$$A_O = 169{,}854.5 \text{ m}^2 .$$

The length and width[25] of the avenue are 89.6 m and 25 m respectively, giving it an area of

$$A_a = 2239.7 \text{ m}^2 ,$$

and the area of the Observatory Circle is

$$A_c = 2239.7 \text{ m}^2 ,$$

so twice the sum of the areas of the Observatory Circle and avenue is

$$2(A_O + A_a) = 166{,}597.9 \text{ m}^2 .$$

The difference between this area and that of the Octagon is 3256.5 m^2.

But recall the smaller circular enclosure of diameter $^1/_7$ OCD which is adjacent to the Octagon. Its area is 1654.2 m^2; twice this figure is 3308.5 m^2. Thus, the area of the Octagon appears to double the combined area of the Observatory Circle, the connecting avenue, *and the small circle* within 51.9 m^2, an error of 0.03%. (Figure 9b).

A closer examination of the Octagon suggests how this relationship might have come about if the Hopewell knew the ratio of a circle's circumference to its diameter: Notice first that

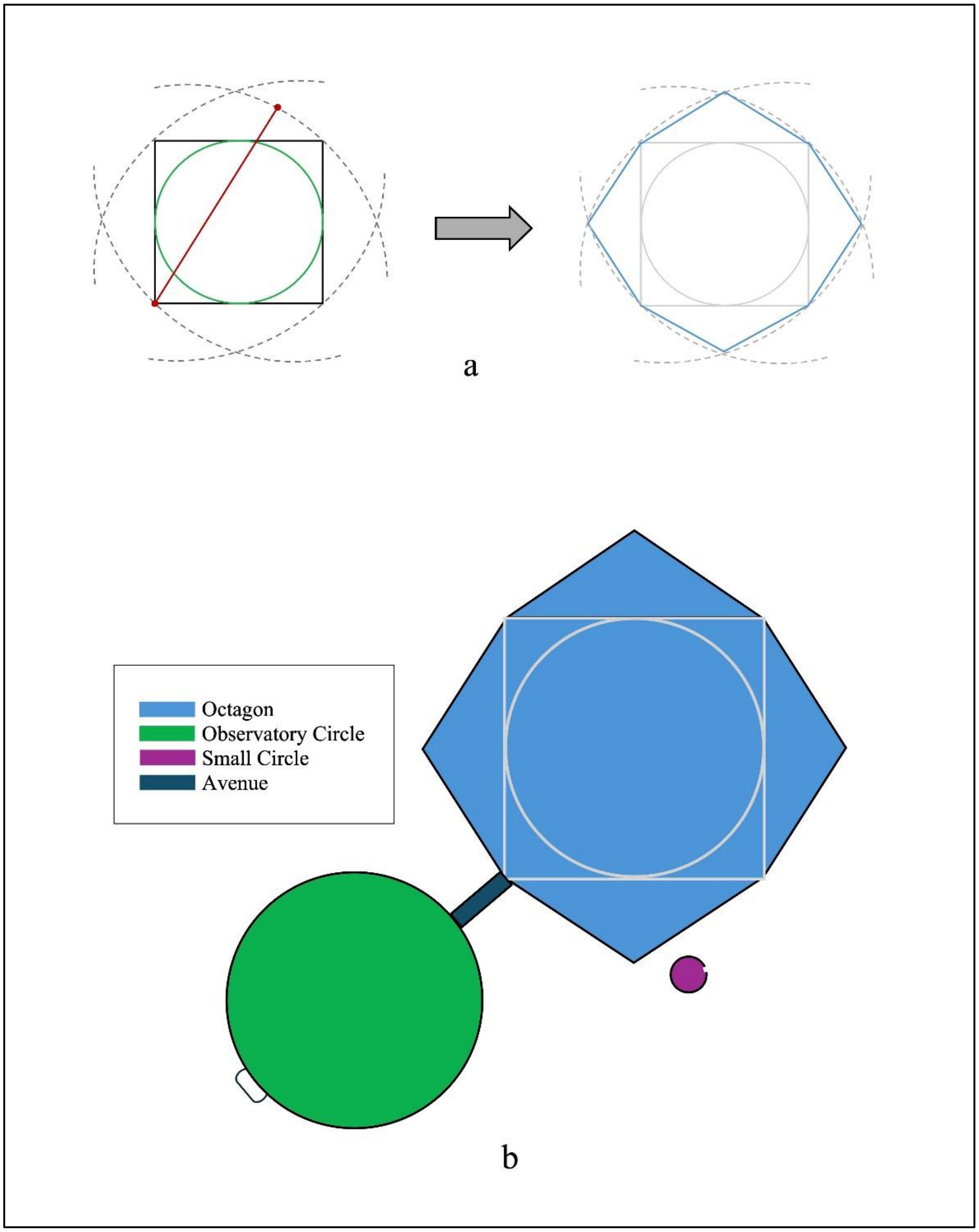

Figure 9. a, Constructing the Octagon; b, the area of the Octagon is twice the combined area of the Observatory Circle, the connecting avenue, and the small circle.

our computation (and conception) of the area of the Octagon can be simplified considerably merely by moving the triangular sections from two sides of our construction to the other two sides, as in Figure 10a.

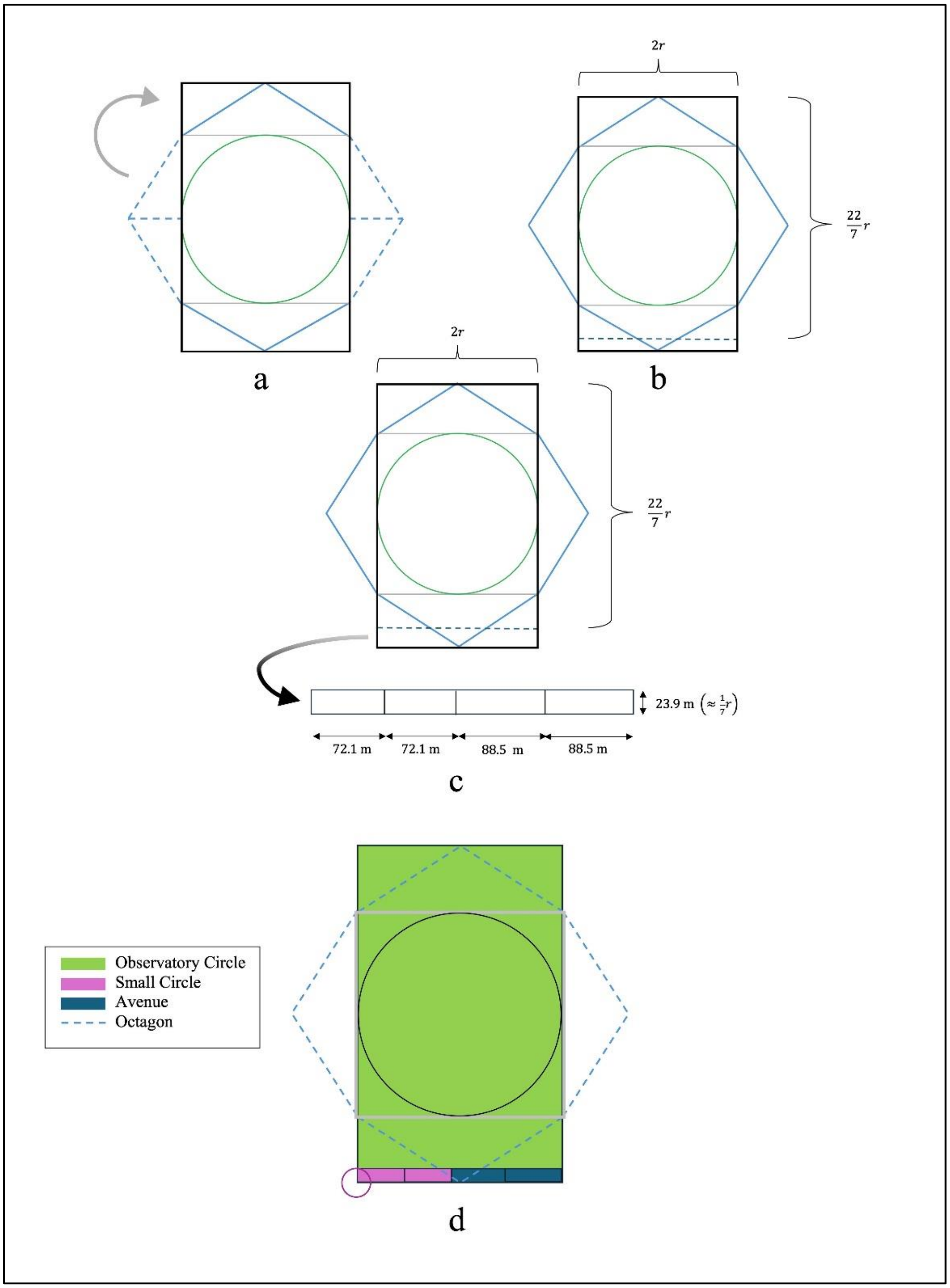

Figure 10. a, Converting the Octagon into a rectangle; b, partitioning the rectangle made from the Octagon; c, partition of the remaining strip; d, the rectangle divided into areas equal to those of the Observatory Circle, connecting avenue, and small circle.

This gives us a rectangle with the same length as the original figure and with a width of $2r$, or twice the radius of the Observatory Circle. If one then marks off a distance equal to $^{22}/_{7}\,r$ from one end of the rectangle and then divides the rectangle at that point, the larger of the two parts will be a rectangle with width $2r$ and length $^{22}/_{7}\,r$, with an area twice that of the Observatory Circle (Figure 10b).

The area of the remaining rectangular strip must therefore be very close to twice the combined areas of the avenue and the small circle. The width of this strip is also very close to the radius of the small circle (23.9 m vs. 22.9 m, respectfully), so that if we measure off two lengths equal to $^{22}/_{7}$ times the small circle's radius (72.1 m), we will have two rectangles, each with very nearly the same area as the small circle. Notice, too, that if we cut the remaining section in half, we will have two rectangles whose length and width are each about 1.1 m less than that of the avenue that connects the Observatory Circle and Octagon (Figure 10c)

The rationale for this method of apportioning the area of the Octagon might be as follows: Assume that we wished to represent the area of the Octagon in terms of circles (in this case the Observatory Circle). One way of doing this is to think of the process as a succession of division problems. We first partition off a rectangular area equal to twice the area of the Observatory Circle, essentially *dividing* the area of the Octagon by that of the Observatory Circle (we might say that the Circle goes into the Octagon twice). The narrow rectangle that is left over has a width approximately equal to one-seventh the radius of the Observatory Circle ($^{1}/_{7}\,r$). We then partition this rectangle by measuring two distances along its length equal to $^{22}/_{7}$ times $^{1}/_{7}\,r$. This gives us two smaller rectangles which we can translate quite easily into circles of the same area. What is then left, the final *remainder* in this division problem, may be similarly partitioned into two rectangles which are each dimensionally similar to the avenue (Figure 10d). Such an explanation is possible if either the diameter of the small circle or the dimensions of the avenue were not otherwise predetermined.

Conclusions

The evidence from historical and contemporary surveys leaves little doubt that equality relationships of area and perimeter exist between the square and circles of the Newark Earthworks. Further, a measurement system together with a simple integral ratio, both reportedly found at Newark and other Hopewell sites, would have greatly aided in their construction. And finally, the decomposition of the area of the octagon into twice the sum of the areas of its neighboring enclosures argues strongly for the intentionality of the area relationships between the figures as well as for a considerable amount of mathematical skill and ingenuity on the part of their builders.

But while some early old-world civilizations possessed similar mathematical knowledge, several things set the Hopewell apart. The fact that they somehow managed to combine their geometrical and astronomical understanding and align their complex constructions with celestial events is certainly a remarkable achievement. But perhaps the most striking difference is that rather than record their discoveries on stone or parchment, the Hopewell chose to express them in the form of colossal earthen mounds. Moreover, these structures were executed with an extraordinary degree of precision, over rough terrain and with only primitive tools, and at sites great distances from one another. And to accomplish this, thousands of people must have worked in concert over a period of years, digging up, transporting and strategically placing tens of millions of baskets of soil, all for some vision or purpose about which we know nothing. So, as we marvel at the secrets we find hidden in the earthworks, we should not neglect the organizational and engineering skills that made them possible.

Acknowledgements
Thanks to Dr. Bradley Lepper, Dr. Ray Hively, Dr. Robert Horn, Dr. Richard Shiels, Steve Williams, Dr. Paul Sciulli, Dr. Guido Mislin, Dr. Barbara Mann and Dr. Carsten Haese for their advice, encouragement, and patience.

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Endnotes

[1] The earliest recorded statement of the problem is found in the Rhind papyrus, which dates from around 1650 B.C., and is a copy of a work written some two centuries earlier (and which may, in turn, be a copy of a much older work).

[2] A recently built highway cuts through the heart of what was once the square. The existing fragment has been extensively restored. Gerard Fowke, who participated in the Smithsonian survey, said of a later (1893-1896) restoration of two of the other enclosures that:

> "…the State has acquired circle F [the Observatory Circle] and the octagon. It is probable that none of these will ever suffer any diminution in size. In fact, the State authorities have a little overdone the matter of

> restoration. Unless there is considerable reduction, from weathering, of that portion which has lately been built up, visitors in generations to come will infer that some parts were originally heavier than others, when such was not the case." (Fowke 1902:171) (brackets mine).

On the other hand, Hively and Horn consider it "unlikely that the State restoration in 1893 could have altered their configuration significantly" (Hively and Horn 1982:S6). Until very recently, the land on which the Observatory Circle and Octagon enclosures are situated was leased to a private country club and used as a golf course. The other circle (the Great Circle) has been, among its many uses, home to the Licking County Fair, a racetrack, and a community park.

[3] Squire and Davis relied in part on the maps and surveys of geologist Charles Whittlesey, whose numbers, despite being "corrected and verified by careful resurveys and admeasurements" (Squier and Davis 1848:67) are sometimes wildly off the mark (Thomas 1889:5-15).

[4] This square is not among the remaining enclosures at the Newark works; however (as will be shown later) it is implicit in the construction of the Octagon: The square may be formed by connecting four of the Octagon's vertices. Put another way, the Octagon circumscribes the square which in turn circumscribes the Observatory Circle (Hively and Horn 1982:S8). An 1862 map of the Newark works from a survey by James and Charles Salisbury shows the existence of a second square (Lepper 1998:4-5), whose reported dimensions are quite close to those of the square that would be inscribed *within* the Observatory Circle, but this appears to be the sole occurrence of it in the historic literature. Romain (2000:39, 62-3) cites examples of squares with sides the length of the Observatory Circle Diameter (OCD) at other Hopewell sites, although these appear to be derived from Squier's and Davis's maps, whose dimensions he acknowledges are unreliable.

[5] The first of these assertions, that the Observatory Circle and the Newark Square have equal areas, is explicit in the 1836 drawing by Charles Whittlesey which accompanies Squier's and Davis's narrative: The correct acreage (20) enclosed by each of the earthworks is clearly marked (but this may be an anomaly. See note 3). To the best of my knowledge, the initial appearance in the modern literature of the first three observations occurs in a footnote in Hively's and Horn's 1982 paper, that of the fourth in Romain (2000:40).

[6] "Measuring the various diameters, the maximum is found to be 1,059 feet [322.8 m] and the minimum 1,050 [320 m], the mean of which is 1,054.5 feet [321.4 m], but it is found by trial that the nearest approximate circle has a diameter of 1,054 feet [321.3 m]. The widest divergence between the line of the survey and the circumference of the true circle is 4 feet [1.3 m]." (Thomas 1894:464) (brackets are mine). *Please note: Unless otherwise noted, all the surveyed dimensions given in this paper are metric conversions of Thomas's measurements. There are several instances where comparisons are made in which the reader may find a discrepancy of 0.1 m due to rounding error. Thomas's measurements were made in feet, with an assumed margin of error of ±6 inches, so anything less than 0.2 m should not be considered significant.*

[7] Hively and Horn (1982:S5-6) note that the results of their survey, as well as those conducted by William Holmes in 1892 and John Eddy in 1978, agree substantially with Middleton's findings, as do Marshall's (1987).

[8] See note 11.

[9] One corner was missing and plowing had severely eroded the entire figure at the time of the Bureau's survey (Hively and Horn arrived about one hundred years too late to conduct one of their own and thus relied on Middleton's survey of the Square for their study). Thomas says of the square: "As it is well-nigh obliterated it was found impossible to trace the lines throughout, hence only those parts are marked in the figure (see Pl. xxxiv) which were satisfactorily determined; the untraced portions are represented by dotted lines." (Thomas 1889:465). (Note: The estimated length of the southeastern wall of the Square is given in Thomas as 951 ft instead of 931 ft. This is evidently a typographical error.)

[10] For example, the angle of intersection of two of the diagonals of the Octagon deviates a mere ten minutes from 90 degrees, while that of the other two is off by only two minutes (Hively and Horn 1982:S8; Thomas 1894:465).

[11] Dr. Paul Sciulli of Ohio State University suggests that both the "flattened" section of the Great Circle and the slight irregularity of the Newark Square might be the result of an attempt to reconcile the circumference-perimeter and area relationships; that is, to accommodate the value determined for the common length of their perimeters to the separately derived linear dimensions of the square and circular enclosures of equal area (personal communication, 2004). Similarly, Hively and Horn (1982) hypothesize that the walls of the Octagon may have been altered to achieve the desired lunar alignments.

[12] Some have suggested that ropes constructed from deer hide. James Kingery, curator of the Newark Earthworks for the Ohio Historical Society, hypothesizes that the cordage might have been made from the leaves of the cattail plant because of their relative strength and inelasticity (personal communication, 2002).

[13] The explanation goes roughly like this: Numbers may be divided into two types, *algebraic* and *transcendental*. A number is algebraic if it is a root (solution) of a polynomial equation with rational coefficients. For example, the number 3 is algebraic because it can be shown to be a root of the equation $3x = 9$; similarly, ½ is algebraic because it is a solution of $4y^2 + 6y = 4$. $\sqrt{2}$ is also algebraic because it is a root of the equation $z^2 = 2$. It turns out that only lengths which can be expressed in terms of algebraic numbers are constructible with straightedge and compass alone. In the examples cited here, it is quite easy to triple or halve the length of a given line; and if a square is constructed with a side of length l, a line connecting two of its opposite corners (the diagonal of the square) will be $l\sqrt{2}$ units long. π, however, is transcendental, which simply means that there is no such equation which has π as a solution, and consequently no such constructible line. It was not until A.D. 1882, some 2200 years after the problem was posed by the Greeks (and a similar period from the first appearance of the Hopewell) that German mathematician Ferdinand von Lindemann showed π to be transcendental, thus ending two millennia of frustration (Boyer 1968:573).

[14] Dr. Bradley Lepper, who participated in a 1992 excavation of a section of the Great Circle at Newark, states that "no one has found any post molds associated with the walls at Newark except for the individual post mold we found in the gravel-filled pit at the gateway of the octagon (and infer for the other gateways). This does not mean that there were no posts. We have barely looked at the Newark walls and the one location we investigated at the Great Circle had been eroded (to an unknown extent) and restored. There originally may have been post molds from a palisade on top, but erosion might have eliminated the evidence. We found no evidence of post molds, however, beneath the section of wall we excavated. If they were widely spaced, we could have simply missed them." (Personal communication, 2004). Cowan and Sunderhaus (2002), in their investigation of the Stubbs Earthworks complex in Warren County, Ohio, discovered a circle 73 meters in diameter containing 172 post holes, at a spot where Whittlesey in an 1851 map had indicated the existence of a circular enclosure. Radiocarbon dating of the fill material found in the holes indicates that the site may have been contemporaneous with the Newark Works. More recently, using more modern techniques, post holes have been found at both the Seip and Hopewell Mound Group sites (Komp, Lüth, et. al.:2020; Ruby: 2020).

[15] The choice of 10 meters here is arbitrary. In the case of the Observatory Circle, for example, the inscribed polygon would have 101 sides, and its perimeter would approximate the circumference within about 0.6 meters (0.06%). In general, the smaller the distance between the posts (and the greater the number of sides), the closer the length of the perimeter will approximate that of the circumference, but with a margin of error of half the distance between the posts.

[16] 22/7 (to four decimal places) is 3.1429, versus 3.1416 for π, a difference of thirteen ten-thousandths.

[17] 6 OCD is also the distance between the centers of the Octagon and Newark Square and appears at the High Bank earthworks and several other sites as well (Hively and Horn 2020:126-31).

[18] But if lengths which are multiples of π are permitted, a solution can be found quite accurately and efficiently with any of several constructions commonly used in elementary geometry texts to find the geometric mean of two lengths.

[19] The illustration in Figure 7 appears in many texts, specifically in Beckmann (1971:18).

[20] Within about 17 cm. Four of these units would total 321.9 m, a difference of 0.2 per cent.

[21] Marshall should not be blamed for this notation; ***u*** and ***u_d*** are mine.

[22] Notably at Piketon and in his reconstruction of the plan of the now obliterated Circleville works (Marshall 1987:43-4). At Circleville, the diameters of the Observatory Circle and Great Circle are repeated, but in this instance the ratio of $\sqrt{\pi}/2$ occurs in circles which are concentric.

[23] This is precisely the result given in the Rhind papyrus (van der Waerden 1983:172). Gerdes (2003:154-62), in his discussion of the geometry of ancient Egypt and Mesopotamia, shows how the 8:9 ratio could have been discovered in the context of basket weaving. Interestingly, the ancient Egyptians arrived at their solution by cutting off the corners of a square to form an octagon (Heilbron 1998:246; van der Waerden 1983:171). Marshall (1987:45) states that this ratio also occurs in the major and minor diameters of an ellipse-like enclosure (which he refers to as a "draftsman's ellipse") at the Liberty Township works. It may be worth mentioning that the large open figure in Squier and Davis's plan of the Newark Works (Figure 2) is a fragment of an elliptically shaped enclosure shown whole on earlier maps of the site.

[24] With the creation of the two area relationships, the perimeter-circumference correspondence between the Great Circle and the Newark Square is implicit. To see this, let r_g represent the radius of the Great Circle, r_o the radius of the Observatory Circle, and s_n the side of the Newark Square. The length of the side of the square which would circumscribe the Observatory Circle is equal to the diameter of the circle, or $2r_o$; consequently, if the area of the Great Circle were equal to the area of the circumscribing square, we would have $\pi {r_g}^2 = (2r_o)^2 = 4{r_o}^2$. Similarly, if the area of the Observatory Circle were equal to the area of the Newark Square, we would have $\pi {r_o}^2 = {s_n}^2$ or ${r_o}^2 = \frac{{s_n}^2}{\pi}$. Replacing r_o with $\frac{{s_n}^2}{\pi}$ in the first equation gives us $\pi {r_g}^2 = 4\left(\frac{{s_n}^2}{\pi}\right)$. Simplifying this: $\pi^2 {r_g}^2 = 4{s_n}^2$ and then taking the square root of both sides, we have $\pi r_g = 2s_n$. The circumference c_g of the Great Circle is then $c_g = 2\pi r_g = 2(2s_n) = 4s_n$ and $4s_n$ is the perimeter of the Newark Square.

[25] The figure of 89.6 m given as the length of the connecting avenue is the average of Hively's and Horn's measurements of its two walls, 89.3 m and 89.9 m.